EFFECT OF INITIAL TEMPERATURE ON THE THICKNESS OF THE UNBURNT LAYER OF POWDER ON A METAL PLATE

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A method for analyzing unsteady combustion of explosives and powders (method of combustion zone "freezing"), based on the assumption of purely thermal one-dimensional effects in the combustion process, was proposed in [1]. Owing to the high thermal conductivity of metal, the heat flux from the combustion zone increases with the approach of the combustion wave to the powder–metal interface. When a certain critical value of the flux is reached, combustion ceases and a layer of unburnt powder remains on the metal surface. Dependance of the thickness of the unburnt powder layer on pressure was theoretically analyzed in [2] and experimentally established in [1]. A quantitative analysis of conditions of powder combustion extinction resulting from the interaction between the combustion zone and the powder–metal interface is given in [3].

Data on the effect of initial temperature on the thickness of the unburnt powder layer derived by the method of "freezing" the combustion zone are presented below. The practicability of approximate calculation of unsteady processes occuring during the approach of a combustion wave to the powder–metal interface is examined. Theoretical and experimental data are compared.

1. Statement of the Model Problem. The problem of thermal interaction between the combustion front and the metal–powder interface for the model of unstable powder combustion proposed by Zel'dovich [4, 5] can be presented in the form [2]

$$\vartheta^{\cdot} = \vartheta'', \qquad 0 < \xi < \xi_s(\tau) \tag{1.1}$$

$$\vartheta(\xi_s, \tau) = 1, \quad \vartheta(0, \tau) = e^{-L}, \quad \vartheta(\xi, 0) = e^{\xi - L}, \quad \xi_s(0) = L \tag{1.2}$$

$$\xi_s^{\cdot} = -w, \qquad w = F(w, (\vartheta')_s) \tag{1.3}$$

The following notation is used:

$$\vartheta(\xi, \tau) = \frac{T(x, t) - T_0}{T_s - T_0}, \quad \xi = \frac{u_0}{\varkappa} x, \quad \xi_s = \frac{u_0}{\varkappa} x_s, \quad \tau = \frac{t u_0^2}{\varkappa},$$
$$L = \frac{u_0 l}{\varkappa}, \quad w = \frac{u}{u_0}$$

Here x and t are, respectively, the coordinate and the time, $\varkappa$ is the coefficient of thermal diffusivity, u_0 and u are the rates of powder combustion under steady and unsteady conditions, respectively, x_s is the coordinate of the moving combustion surface, l is the thickness of the powder layer at the initial instant of time, T_s is the temperature at the surface of combustion, and T_0 is the initial temperature of the powder. Dots and primes denote differentiation with respect to time and coordinate, respectively.

The boundary condition at the powder–metal interface ($\xi = 0$), introduced in [3], avoids the difficulties related to initial conditions at infinity (the so-called degenerated boundary conditions) of the form

$$\vartheta(\xi, 0) \to \exp(\xi - \xi_s), \qquad \xi_s \to \infty$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 11, No. 2, pp. 85–89, March-April, 1970. Original article submitted August 12, 1969.

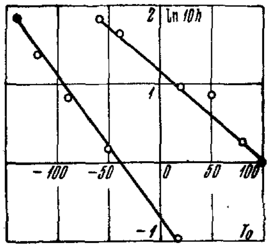

Fig. 1

The error introduced by the substitution of condition (1.2) for the zero boundary condition at the interface can be made as small as desired by choosing a sufficiently thick initial layer of the powder sample ($L \gg 1$).

If the dependence of the stable rate of combustion on temperature T_0 and pressure p is approximated by formula $u_0 = u_1 p^\nu \exp \beta T_0$, the function $F(w, (\vartheta')_s)$ defining the relation between the unstable combustion rate and the temperature gradient at the surface of combustion is of the form

$$w = \exp\left[1 - \frac{1}{w}(\vartheta')_s\right], \qquad \varepsilon = \beta(T_s - T_0), \qquad \beta = \left(\frac{\partial \ln u_0}{\partial T_0}\right)_p \tag{1.4}$$

If, however, this dependence is approximated by function $u_0 = ap^\nu/(1 - \alpha T_0)$, then for function $F(w(\vartheta')_s)$ we find

$$w = \frac{1 - \varepsilon(\vartheta')_s}{1 + \varepsilon} \tag{1.5}$$

2. The Approximate Solution. An explicit analytical solution of the problem (1.1)-(1.3) is not possible. The most widely used method for the approximate solution of problems of unstable combustion is that of integral relationships [6-9]. The success of this method largely depends on a correct a priori selection of the form of the sought approximate solution. Let us investigate the possibility of solving problem (1.1)-(1.3) by expressing the dimensionless temperature in the form

$$\vartheta(\xi, \tau) = e^{\xi - \xi_s} - \left(1 - \frac{\xi}{\xi_s}\right)(e^{-\xi_s} - e^{-L}) + \left(1 - \frac{\xi}{\xi_s}\right)^2 \sum_{n=0}^{\infty} a_n \left(\frac{\xi}{\xi_s}\right)^n \tag{2.1}$$

Here coefficients $a_n = a_n(\tau)$ are the unknown functions of time such that $a_n(0) = 0$.

Function (2.1) satisfies the boundary and initial conditions (1.2). For a more specific definition of the approximate solution, it is necessary to write down the equations determining functions $a_n(\tau)$. A satisfactory approximation can be hoped for, if in expansion (2.1) we retain only the term with $n = 0$ and define function $a_0(\tau)$ by the heat-balance integral [8]

$$\frac{d}{d\tau} \int_0^{\xi_s} \vartheta(\xi, \tau)\, d\xi - \dot{\xi}_s = (\vartheta')_s - (\vartheta')_0 \tag{2.2}$$

Substituting (2.1) into (2.2), we find

$$\xi_s^2 \dot{a}_0 + (\dot{\xi}_s \xi_s - 6) a_0 + {}^3/_2\, \xi_s \dot{\xi}_s (e^{-\xi_s} + e^{-L} + \xi_s e^{-\xi_s} - 2) - 3\xi_s (1 - e^{-\xi_s}) = 0 \tag{2.3}$$

Equation (1.3), which determines the variation of the ξ_s-coordinate of the surface of combustion, now takes the form

$$\dot{\xi}_s = -F(\dot{\xi}_s, (\vartheta')_s) \tag{2.4}$$

$$(\vartheta')_s = 1 + (e^{-\xi_s} - e^{-L})/\xi_s \tag{2.5}$$

Note that function (2.1) is such that the temperature gradient at the surface of combustion is expressed in terms of only the ξ_s-coordinate of that surface; hence, the approximate equation for the rate of unstable combustion can be integrated independently of Eq. (2.3).

Thus, when the relationship between the rate of unstable combustion and the temperature gradient at the surface of combustion is defined by (1.5), Eq. (2.4) is integrable in quadratics, which yields

$$\tau = \int_L^{\xi_s} \frac{x\,dx}{Px - Qe^{-x}}, \qquad P = \frac{1 - \varepsilon}{1 + \varepsilon}, \qquad Q = \frac{\varepsilon}{1 + \varepsilon}$$

Let us examine the problem of determining the thickness h of the layer of powder which remains unburnt on the surface of metal with (1.4) as the law of combustion extinction proposed by Zel'dovich. According to [4, 5], the temperature gradient at the surface of combustion cannot exceed, under given conditions, a certain maximum value equal to

$$(\vartheta')_s^* = \frac{1}{\varepsilon} e^{\varepsilon - 1} \tag{2.6}$$

Equating the expression (2.5) of the temperature gradient at the surface of combustion to its critical value (2.6), we obtain

$$\frac{1}{\varepsilon}e^{\varepsilon-1} = 1 + \frac{1}{\Delta}(e^{-\Delta} - e^{-L}), \qquad \Delta = \frac{hu_0}{\varkappa} \tag{2.7}$$

which determines the thickness of the powder layer on the metal at the instant of combustion extinction.

Parameter e^{-L} can, obviously, be neglected when actually solving Eq. (2.6). This equation has a solution for any value of parameter ε.

Equation (2.7) makes it possible to investigate the dependence of h on the initial temperature T_0 and pressure p. Differentiating (2.7) with respect to T_0 and p, we obtain

$$\left(\frac{\partial h}{\partial p}\right)_{T_0} = -\nu\frac{h}{p} + \frac{1-\varepsilon}{\varepsilon^2}e^{\varepsilon-1}\left[\frac{1+\Delta}{\Delta}e^{-\Delta}\right]^{-1}\frac{\beta\varkappa}{u_0}\left(\frac{\partial T_s}{\partial p}\right)_{T_0} \tag{2.8}$$

$$\frac{1}{\beta}\left(\frac{\partial \ln h}{\partial T_0}\right)_p = -1 - \frac{1-\varepsilon}{\varepsilon^2}e^{\varepsilon-1}\left[\frac{1+\Delta}{\Delta^2}e^{-\Delta}\right]^{-1}\frac{1}{\Delta} \tag{2.9}$$

In the particular case of $(\partial T_s/\partial p)_{T_0} = 0$, Eq. (2.8) yields the dependence of the thickness of unburnt powder layer on pressure, established earlier by means of similarity and dimensional analysis in [2].

Equation (2.9) shows that the thickness of the unburnt residual layer decreases with increasing initial temperature.

Let us consider the approximate solution of the problem of interaction between the combustion zone and the metal–powder interface using the model of a combustion zone with variable surface temperature [10]. For this we have to use in the input equations (1.1)-(1.3), when passing to dimensionless coordinates, the value of T_s prevailing at the initial instant of time, and substitute condition $\vartheta(\xi_s, \tau) = \Phi((\vartheta')_s)$, where Φ is a known function of the temperature gradient at the surface of combustion, for the conditions at that surface. The form of the a priori expected solution, continuously tending to the Michelson profile, can then be written

$$\vartheta(\xi, \tau) = \Phi((\vartheta')_s)e^{\xi-\xi_s} - \left(1 - \frac{\xi}{\xi_s}\right)(e^{-\xi_s} - e^{-L}) + \left(1 - \frac{\xi}{\xi_s}\right)^2\sum_{n=0}^{\infty}a_n\left(\frac{\xi}{\xi_s}\right)^n \tag{2.10}$$

Function Φ, obviously, satisfies the initial condition $\Phi(1) = 1$. For the temperature gradient at the surface of combustion, from (2.10) we obtain

$$(\vartheta')_s = \Phi((\vartheta')_s + \frac{1}{\xi_s}(e^{-\xi_s} - e^{-L}) \tag{2.11}$$

Let us use this formula for investigating the motion of the representative point in the diagram of $w(\vartheta')_s$.

Differentiating (2.11) and function $w = w((\vartheta')_s)$, we obtain

$$b^{\cdot} = \frac{w}{1 - \frac{d\Phi}{db}}\frac{1}{\xi_s^2}[e^{-\xi_s} + \xi_s(e^{-\xi_s} - e^{-L})], \quad w^{\cdot} = \frac{dw}{db}b^{\cdot}, \quad b \equiv (\vartheta')_s \tag{2.12}$$

According to [10],

$$\frac{d\Phi}{db} = \frac{r}{r+\varepsilon-1}, \quad \frac{dw}{db} = \frac{\varepsilon}{r+\varepsilon-1}, \quad r = \frac{dT_s}{dT_0}$$

From (2.12) it follows that

$$w^{\cdot} = \frac{\varepsilon w}{\varepsilon - 1}\frac{1}{\xi_s^2}[e^{-\xi_s} + \xi_s(e^{-\xi_s} - e^{-L})] \tag{2.13}$$

The derived relationship is valid not only for functions of the kinds (1.4) and (1.5), but also for the relationships analyzed in [3, 10]. Equation (2.13) shows that with the approach of the combustion front to the metal–powder interface, the combustion rate increases when $\varepsilon > 1$ and decreases when $\varepsilon < 1$, independently of the magnitude of parameter r.

3. The Experiment. Investigations of the effect of initial temperature of the specimen on the thickness of the unburnt powder layer on a copper plate were carried out in a nitrogen atmosphere in a constant pressure vessel. A mark-H nitroglycerin powder was chosen for this investigation. In the first series of tests at a pressure of 1 atm, specimens of 23.6 and 9 mm diameter were used, while in the second, carried out at a pressure of 20 atm, 6-mm-diam. specimens were used. The results are as follows:

at the pressure of 1 atm,

T_0 (°C) =	−60	−40	20	50	80	100°
h(mm) =	0.62±0.02	0.52±0.06	0.27±0.01	0.24±0.02	0.13±0.01	0.10±0.01

at the pressure of 20 atm,

T_0 (°C) =	−140	−120	−90	−50	18
(mm) =	0.65±0.07	0.40±0.03	0.23±0.05	0.12±0.01	0.04

Within the limits of accuracy of this experiment the derived dependence plotted in semi-logarithmic coordinates can be considered as linear (see diagram). The slope of these straight lines is

$$\Delta \ln h / \Delta T_0 = -11.5 \cdot 10^{-3} \ (\mathrm{deg})^{-1} \text{ for } p = 1 \text{ at}$$
$$\Delta \ln h / \Delta T_0 = -15.2 \cdot 10^{-3} \ (\mathrm{deg})^{-1} \text{ for } p = 20 \text{ at}$$

It is interesting to recall in this connection that according to [11] the temperature coefficient of the rate of combustion for the mark-H powder varies as follows:

at 20 atm,

T_0 (°C) =	−150	−100	−50	0	50	100	140
β (°C^{-1}) =	0.5	2.0	3.5	5.0	7.0	9	15

at 1 atm,

T_0 (°C) =	−200	−100	0	50	100
β (°C^{-1}) =	1.0	3.0	9.8	12.5	14.3

At the same time, according to [12] the coefficient β can be considered to be a piecewise-constant function: for $T_0 < 20$°C, $\beta = 1.95 \times 10^{-3}$ (deg)$^{-1}$; and for $T_0 > 20$°C, $\beta = 14.6 \times 10^{-3}$ (deg)$^{-1}$

A qualitative correlation of obtained theoretical and experimental results can be seen. The dependence of the thickness of the unburnt powder layer on a metal surface on initial temperature − the decreasing function of temperature $d \ln h/dT_0 < 0$ − is confirmed. The experimentally obtained value of $d \ln h/dT_0$ is close as to its order of magnitude to the temperature coefficient of combustion rate β. It should be noted that a quantitative comparison of theoretical and experimental data on the relationship $h(T_0)$ is not possible for the following reasons. The application of the Zel'dovich theory of powder combustion [4, 5], and, in particular, of the criterion of combustion extinction, to the mark-H powder would mean going beyond the limits of applicability of that theory. The limits of stability of the stable combustion mode of the mark-H powder are noticeably wider than theoretical estimates. There is an anomalous effect of the initial temperature on the rate of steady combustion and, as a consequence of this, the dependence of the unstable combustion rate on the temperature gradient at the surface of combustion differs from that considered by Zel'dovich in [4, 5]. The criteria of combustion extinction proposed in [3, 10] for the mark-H powder cannot be at present computed.

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